

Grade-Level Standard(s)	Thumbnail Sketch of A Single-Algorithm Approach to Meeting These Grade-Level Standards (feedback welcome)
<p>1.NBT.4. Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten.</p>	<p>This grade 1 standard can be met by teaching two-digit addition in first grade in roughly the following way.</p> <p>* Before we get started, let’s note key prerequisites for adding two-digit numbers with the standard algorithm:</p> <ul style="list-style-type: none"> - Students should know the quantities that the numbers 1–20 name. These and other foundations are in the K standards. - Students should be fluent with all sums less than ten (such as $3 + 5$) and all sums equaling ten (such as $6 + 4$), as well as some or all of the sums that “cross ten,” such as $6 + 6$ or $9 + 4$. Therefore, from the beginning of the school year in grade 1, one would want to invest strongly in building fluency with sums less than or equal to 10; one would also want to build fluency gradually with sums that “cross ten.” (Related differences can also be included in fluency work according to the usual “number bond” approach; I mention sums here specifically only because subtraction isn’t strictly a prerequisite for two-digit addition. Note, finally, that knowing all single-digit sums from memory is an end-of-grade-2 expectation.) - Other prerequisites include basic elements of place value: that the “decade numbers,” written $A0$, amount to A tens; that 100 equals 10 tens; that the teen numbers, written $1B$, amount to 1 ten and B ones; and that a two-digit number, written AB, amounts to A tens and B ones. However, to call this material “prerequisite” is simplistic; working on two-digit addition also reinforces and reteaches these concepts. - Given all this, it is likely that the proper time for taking on general two-digit addition directly is during the second half of the school year. <p>* When the time is right, one would teach students the following procedure for adding two two-digit numbers:</p> <ol style="list-style-type: none"> (1) Write the two addends, one above the other, aligning the places. (Why will the result be wrong if the places aren’t aligned?) (2) It doesn’t matter which number goes on top and which goes on bottom. (Why doesn’t it matter?) (3) Add right to left. (For written computations this turns out to be efficient, as could be illustrated.) (4) Regroup as necessary (what parents call “carrying the 1”; naturally, this move rests on place value concepts). <p>Note that in the procedure being specified here, a horizontal line runs beneath the bottom addend, and the digits of the sum are written immediately below this line.</p> <p>This is the standard algorithm as it exists at this grade level. One would do whatever teaching is necessary, during and beforehand, in order to make this a logical procedure for students, and sequence the work appropriately (e.g., problems like $42 + 35$ before problems like $46 + 38$). One would also give enough practice over time so that students are comfortable carrying out the steps.</p> <p>* Also present non-generic problems that are probably better solved mentally than in writing, like $40 + 8$ or $30 + 20$. There is a role for this kind of work before, during, and after learning the algorithm.</p> <p>In summary, the curricular goals for two-digit addition in grade 1 could be something like the following:</p> <ol style="list-style-type: none"> (1) To add any two two-digit numbers using the standard algorithm (2) To occasionally <u>check</u> such a calculation using concepts of place value and properties of operations—for example, by using mental arithmetic, by writing equations, or by drawing pictures of tens and ones (3) To add mentally in sufficiently simple cases, such as $40 + 8$ or $50 + 20$

Grade-Level Standard(s)	Thumbnail Sketch of A Single-Algorithm Approach to Meeting These Grade-Level Standards (feedback welcome)
<p>2.NBT.5. Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.</p> <p>2.NBT.6. Add up to four two-digit numbers using strategies based on place value and properties of operations.</p> <p>2.NBT.7. Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.</p>	<p>These standards can be met by teaching the material in second grade in roughly the following way. (The following points are listed sequentially, but that doesn't necessarily imply finishing one point before beginning the next.)</p> <ul style="list-style-type: none"> * Replicating the grade 1 addition approach for two-digit subtraction, then over the course of the year practicing general two-digit addition and subtraction to fluency. * Extending the standard addition algorithm to the case of three and four addends. (In principle this is easy because the standard algorithm scales so well, but the demands on mental computation and single-digit fluency do become greater with more addends.) * Most importantly, extending the standard addition algorithm to three-digit numbers by replicating the grade 1 approach, and extending the grade 2 two-digit subtraction approach to three-digit numbers. * Incorporating these sums and differences into word problems. * Also showing opportunistic strategies for non-generic computations, for example problems such as $212 - 13$, $100 - 88$, or $80 + 123 + 20$ where the standard algorithm is probably both slower and less reliable than a readily apparent mental strategy. (One wouldn't penalize a student for passing up an opportunity for mental math and carrying out the written algorithm smoothly; but students who consistently pass up accessible opportunities may be having trouble with number concepts and fluency.)
<p>3.NBT.2. Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.</p>	<p>This standard is the same as 2.NBT.7 except for (a) a new expectation of fluency in the three-digit case and (b) the first mention of "algorithms." This standard can be met by simply continuing to practice the grade 2 work to the point of fluency. (It is true that the word "algorithms" here is plural, but that could simply be read as leaving more choice in the hands of the teacher about which algorithm(s) to teach—not as a requirement for each student to learn two or more general algorithms for each operation!)</p>
<p>4.NBT.4. Fluently add and subtract multi-digit whole numbers using the standard algorithm.</p>	<p>This is the culminating standard for multi-digit addition and subtraction—the endpoint of the development being sketched here from a grade 1 starting point.</p> <ul style="list-style-type: none"> * Practicing using the standard algorithm for bare computations of sums and differences with generic multi-digit numbers. (Summative assessments should have tighter limits on the number of digits, but six-digit numbers are OK in the curriculum because they align with the conceptual understanding of place value expected at this grade and better reveal the recursive nature of the algorithm itself.) * Incorporating these generic multi-digit sums and differences into word problems. * Also showing opportunistic strategies for non-generic computations, for example problems such as $6,012 - 13$, $400 - 388$, or $800 + 1,234 + 200$ where the standard algorithm is probably both slower and less reliable than a readily apparent mental strategy. (One wouldn't penalize a student for passing up an opportunity for mental math and carrying out the written algorithm smoothly; but students who consistently pass up accessible opportunities may be having trouble with number concepts and fluency.)